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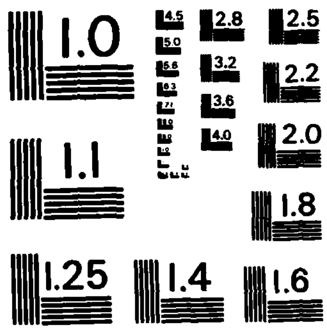

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A method is presented for assigning min(n+q), (q+1)m+q) closed loop poles of an nth order linear time invariant system (m outputs, ℓ inputs, $m \geq \ell$) using a linear, time invariant, proper, output feedback compensator of order q. It is also shown how the locations of remaining unassigned poles could be controlled. The compensator parameters are obtained by solving a linear matrix equation.

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Reference 3

Generic Pole Assignment Using Dynamic Output Feedback

Theodore E. Djaferis*

Abstract

A method is presented for assigning $\min(n+q, (q+1)m+q)$ closed loop poles of an nth order linear time invariant system (m outputs, ℓ inputs, $m \geq \ell$) using a linear, time invariant, proper, output feedback compensator of order q. It is also shown how the locations of remaining unassigned poles could be controlled. The compensator parameters are obtained by solving a linear matrix equation.

*The author is with the Department of Electrical and Computer Engineering at the University of Massachusetts, Amherst, Massachusetts 01003. This research has been supported by NSF under grant ECS-8006896 and partially supported by AFOSR under grant AFOSR-80-0155.

1. Introduction

The problem of assigning the closed loop poles of a linear time invariant multivariable system using a proper, linear, time invariant, output feedback compensator continues to be of great interest. Even though several outstanding issues remain, good progress has been made as evidenced by the interesting work of several researchers. Some recent work can be found in the references.

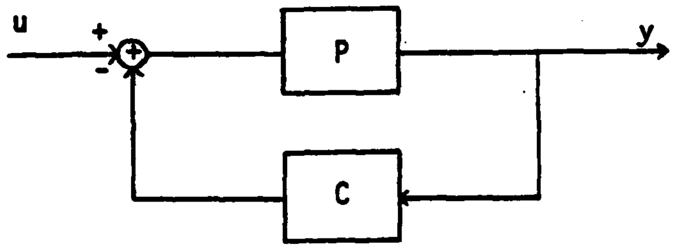
In particular Kimura 1975, Davison and Wang 1975 show that for a controllable observable plant (order n, m outputs, ℓ inputs) it is "almost always" possible to assign $\min(n,m+\ell-1)$ closed loop poles arbitrarily close to a given set of real and complex conjugate values, by using constant output feedback. The issue of what happens to the remaining unassigned poles is not addressed. In a recent paper Antsaklis and Wolovich 1977 present a different way of assigning $\min(n,\ell+m-1)$ poles and suggest ways of dealing with the remaining unassigned poles. They also extend their result to include dynamic output feedback and show that if the original system is initially augmented by q integrators and then constant output feedback applied, $m+\ell+2q-1$ poles can be assigned.

The present work deals with the question of how many poles can be assigned when the order of the compensator is fixed. It is shown that if a compensator of order q is used then $\min(n+q,(q+1)m+q)$ poles can be arbitrarily assigned. Throughout the paper it is assumed that $m \geq \ell$. This is not a restrictive assumption because the $\ell \geq m$ case can be treated in a very similar way and "dual" results obtained, (i.e. $\min(n+q,(q+1)\ell+q$ poles can be assigned). The method of attack is different than the previous ones and it offers an improvement, in the appropriate cases, over the earlier result. It is further suggested how the locations of the remaining unassigned poles can be controlled.

The work proceeds in the following manner. The question of assigning real poles is addressed initially. The Main Lemma takes up the issue with a strictly proper plant with m outputs and one input. This result provides insight as to how the general case might be handled and is successfully applied to the $m \times \ell$, $m \geq \ell$ case in the Theorem. It is then shown that the case of real and complex conjugate poles can be treated in the same manner.

2. Formulation

The following feedback configuration is considered:



where P is a strictly proper $m \times \ell$ input-output transfer function which represents the plant and C some $\ell \times m$ proper dynamic compensator. Both P and C have elements in R(s) the field of rational functions in s over the reals R. The closed loop transfer function G is given by

$$G = P(I + CP)^{-1}.$$

If the following notation for matrix fraction representations (Desoer and Vidyasagar 1975, Kailath 1980) is used:

$$P = B_{RP}A_{RP}^{-1} \qquad \text{some right representation of P,}$$

$$= A_{LP}^{-1}B_{LP} \qquad \text{some left representation of P,}$$

$$= N_{RP}D_{RP}^{-1} \qquad \text{some right coprime representation of P,}$$

$$= D_{LP}^{-1}N_{LP}^{-1} \qquad \text{some left coprime representation of P,}$$

then G can be expressed as:

$$G = B_{RP}(A_{LC}A_{RP} + B_{LC}B_{RP})^{-1}A_{LC}$$

$$= N_{RP}(\underbrace{D_{LC}D_{RP} + N_{LC}N_{RP}}_{\Phi})^{-1}D_{LC} = N_{RP}\ \Phi^{-1}D_{LC}$$

$$= \tilde{N}_{RP}\ \tilde{\Phi}^{-1}\ \tilde{D}_{LC} \qquad \text{(least order)},$$

where $\tilde{N}_{RP}$, $\tilde{\Phi}$ are right coprime, $\tilde{D}_{LC}$, $\tilde{\Phi}$ left coprime.

It can be shown (Chen and Hsu 1968) that if $\phi(s)$ is the characteristic polynomial of the closed loop system then

$$\phi(s) = \alpha \det \Phi$$

where α is some non-zero constant.

Now if $\phi(s) = s^i + \phi_{i-1} s^{i-1} + \phi_{i-2} s^{i-2} + \ldots + \phi_o$ we know that $\underline{s} = (s_1, s_2, \ldots, s_k)$ $k \le i$ are roots of $\phi(s)$ iff

$$[\phi_{i-1}, \phi_{i-2}, \ldots, \phi_0] \underbrace{\begin{bmatrix} s_1^{i-1} & s_2^{i-1} & & s_k^{i-1} \\ s_1^{i-2} & s_1^{i-2} & & s_k^{i-2} \\ \cdot & \cdot & \cdots & \cdot \\ \cdot & \cdot & & \cdot \\ \cdot & \cdot & & \cdot \\ s_1 & s_2 & & s_k \\ 1 & 1 & & 1 \end{bmatrix}}_{Q} = -[s_1^i, s_2^i, \ldots, s_k^i]$$

where Q is an ixk matrix.

Several definitions of genericity have been used. Throughout this paper a set $S \subset R^t$ will be called <u>generic</u> if it contains a non-empty Zariski open set of R^t (Zariski and Samuel 1958).

3. Single-Input, Multiple-Output Case

If P is an mxl strictly proper transfer function, of McMillan degree n, it can be written in the right coprime representation:

$$P = N\, d^{-1}$$

where

$$d = s^n + d_{n-1}\, s^{n-1} + \dots + d_o$$

$$N = N_{n-1}\, s^{n-1} + \dots + N_o\ .$$

Now a 1xm proper compensator of order q can be written in the left representation

$$C = x^{-1}\, Y$$

where

$$x = s^q + x_{q-1}\, s^{q-1} + \dots\ x_o$$

$$Y = Y_q\, s^q + Y_{q-1}\, s^{q-1} + \dots + Y_o$$

If x, Y are left coprime then the closed loop characteristic polynomial is:

$$\phi(s) = xd + YN = s^{n+q} + \phi_{n+q-1}\, s^{n+q-1} + \dots + \phi_o\ .$$

This relationship can be expressed in the following way:

$$[1,\ Y_q,\ x_{q-1},\ Y_{q-1},\ \dots, x_o,\ Y_o]\ S_{q+1}(d,N) = [1, \phi_{n+q-1}, \dots, \phi_o] \tag{3.1}$$

where $S_{q+1}(d,N)$ is the $q+1^{th}$ order Sylvester Resultant (Bitmead et al 1978) of d, N (a $(q+1)(m\times 1) \times (n+q+1)$ matrix).

From the above one can clearly see that the coefficients of the characteristic polynomial are linear functions of the compensator parameters. This consideration will play a crucial role in the $m\times\ell$ case where a compensator structure will be employed, that satisfies this condition.

Main Lemma

Let $P = Nd^{-1}$ be an mx1 strictly proper transfer function, with d of degree n, with $m|n$ and $q \geq 0$ a fixed integer. Let $k = (q+1)m + q \leq n+q = i$, and $\phi(s)$ be the closed loop characteristic polynomial and define:

$$W = \{(N,d) \in R^{mn+n} | d = s^n + d_{n-1}s^{n-1} + \ldots + do, N = N_{n-1}s^{n-1} + \ldots + N_0\}$$

$$S = \{(s_1, s_2, \ldots, s_k) \in R^k | s_i \text{ real}\}$$

$$Z = \left\{(N,d,\underline{s}) \in R^{mn+n} \times R^k \,\middle|\, \begin{array}{l}\text{For which there exists a proper compensator} \\ \text{of order q such that } s_1, s_2, \ldots, s_k \text{ are roots} \\ \text{of } \phi(s)\end{array}\right\}$$

Then Z is a generic subset of $R^{mn+n} \times R^k$.

Remark: The requirement that $m|n$ is not restrictive and it is introduced for convenience. The general case can be treated with very minor alterations.

Let $\overline{S}_{q+1}$, be the submatrix obtained from $S_{q+1}(d,N)$ by removing the first row and first column and α the first row of $S_{q+1}(d,N)$ with the first entry removed.

Since $s_1, s_2, \ldots, s_k$ will be the roots of $\phi(s)$ iff

$$[\phi_{i-1}, \phi_{i-2}, \ldots, \phi_0]\, Q = -\,[s_1{}^i, s_2{}^i, \ldots, s_k{}^i]$$

$s_1, s_2, \ldots, s_k$ will be the roots of $\phi(s)$ if we can find a $\underline{y}$

$$\underline{y} = [Y_q, x_{q-1}, Y_{q-1}, \ldots, x_0, Y_0]$$

such that

$$\underline{y} \cdot \overline{S}_{q+1} \cdot Q = - [s_1{}^i, s_2{}^i, \ldots, s_k{}^i] - \alpha \cdot Q . \qquad (3.2)$$

The compensator which will accomplish this is

$$C = x^{-1} Y ,$$

$$x = s^q + x_{q-1} s^{q-1} + \ldots + x_0 ,$$

$$Y = Y_q s^q + \ldots + Y_o .$$

The Main Lemma suggests that this can be done for "almost all" N, d and $\underline{s} = (s_1, \ldots, s_k)$.

Proof:

It is required to show that Z contains a non-empty Zariski open set. The matrix $\overline{S}_{q+1}$ Q is kxk. Let $F \subset R^{mn+n} \times R^k$ for which it is invertible. $\overline{S}_{q+1}$ is rank k on a Zariski open set, which is non-empty since any Nd^{-1} with equal observability indices will belong to this set (Bitmead 1978). The matrix Q is also rank k for a generic subset of R^k since Q is rank k for any $\underline{s}$ for which the s_i are distinct. The product will be invertible for a generic subset of $R^{mn+n} \times R^k$. Equation (3.2) therefore does have a solution for a generic subset of $R^{mn+n} \times R^k$.

We further need to show that the x(s), Y(s) so constructed are generically left coprime. Clearly x(s), Y(s) are left coprime in a Zariski open set, we just need to show that this is non-empty. To do this we must suggest an $(N,d,\underline{s})$ which can be thought of as a point in $R^{mn+n} \times R^k$ space for which the $x(s)$, $Y(s)$ are left coprime. This is done in two steps:

a) The 1x1 case.

Let $s_1, s_2, \ldots, s_k, s_{k+1}, \ldots, s_i$ be real and distinct with

$s_1 \cdot s_2 \cdot \ldots \cdot s_i > 0$.

Let $d(s) = s^n$.

Let $n_0, n_1, \ldots n_{n-1}$ be defined as follows:

$$(-1)^i n_0 = s_1 \cdot s_2 \cdot \ldots s_i$$
$$(-1)^{i-1} n_1 = \text{sum of all possible products of } i-1 \text{ roots at a time.}$$
$$(-1)^{i-2} n_2 = \text{sum of all possible products of } i-2 \text{ roots at a time.}$$
$$\vdots$$
$$(-1)^{i-n+1} n_{n-1} = \text{sum of all possible products of } i-(n-1) \text{ roots at a time.}$$

The solution of (3.2) then becomes

$$\underline{y} = [Y_q, x_{q-1}, \ldots, x_0, Y_0]$$

where $\quad Y_q = Y_{q-1} = \ldots = Y_1 = 0, \quad Y_0 = 1$

and

$$(-1)^{i-n} x_0 = \text{sum of all possible products of } i-(n-1)-1 \text{ roots at a time}$$
$$\vdots$$
$$(-1)^2 x_{q-2} = \text{sum of all possible products of 2 roots at a time}$$

$$(-1) x_{q-1} = s_1 + s_2 + s_3 + \ldots + s_i \text{ .}$$

An $\underline{s}$ $(s_1 \cdot s_2 \cdot \ldots \cdot s_i > 0)$ can be found such that for this test point $\overline{S}_{q+1}$ Q is full rank and the corresponding $x(s)$, $Y(s)$ left coprime.

b) The mx1 case.

Let $d(s) = s^n + d_{n-1}\, s^{n-1} + \dots + d_o$

$$N(s) = \begin{bmatrix} K_{n-1} \\ n_{n-1} \end{bmatrix} s^{n-1} + \begin{bmatrix} K_{n-2} \\ n_{n-2} \end{bmatrix} s^{n-2} + \dots + \begin{bmatrix} K_o \\ n_o \end{bmatrix}$$

where the K_j's are m-1x1 vectors. The assignment of $d_{n-1},\dots,d_o,\ n_{n-1},\dots,n_o,\ s_1,s_2,\dots,s_i$ is as in part a).

The K_j's are assigned in the following way:

Let $K_o = K_1 = \dots = K_q = 0,\ K_{n-q} = K_{n-q} = \dots = K_{n-1} = 0$

with $t = n-2q-1$ let,

$$K_{q+1} = \bar{K}_1$$
$$K_{q+2} = \bar{K}_2$$
$$\vdots$$
$$K_{q+t} = \bar{K}_t\ .$$

Choose $\bar{K}_1, \bar{K}_2, \dots \bar{K}_t$ and $\underline{s}$ in such a way that

$$\bar{K} = \begin{array}{cc} \begin{bmatrix} \bar{K}_t & \bar{K}_{t-1} & \dots & \bar{K}_1 & 0 & \dots & 0 \\ 0 & \bar{K}_t & \dots & \bar{K}_2 & \bar{K}_1 & \dots & 0 \\ & & & \vdots & & & \\ 0 & 0 \ \dots & \bar{K}_t & \dots & & & \bar{K}_1 \end{bmatrix} & \text{q+1 block rows} \end{array}$$

$\bar{K}$ and $\bar{S}_{q+1}\, Q$ are full rank.

The solution to the corresponding equation 3.2 is:

$$\underline{y} = [Y_q, x_{q-1}, \ldots, x_o, Y_o]$$

$$Y_q = Y_{q-1} = \ldots = Y_1 = 0, \quad Y_o = [0, 0, \ldots, 1]$$

The $x_{q-1}, \ldots, x_o$ as in part a).

And again x(s), Y(s) are left coprime, for this test point.

The above guarantee that x(s), Y(s) are generically left coprime.

This completes the proof of the Main Lemma.

The Main Lemma suggests that for "almost all" mx1 transfer functions of McMillan degree n and for "almost all" $\underline{s}$, k = (q+1)m + q there exists a proper compensator of order q such that $s_1, s_2, \ldots, s_k$ are k roots of the closed loop characteristic polynomial $\phi(s)$. Since $\phi(s)$ is of degree n+q this means that in general there are n-(q+1)m unassigned poles. This means that some of these could even be unstable. As this is a matter of concern it is presently under investigation. It is possible that by restricting the assignable roots to lie in a certain region to assure that all of them are stable. In the general $m\times\ell$ case it is shown how additional compensator parameters can be introduced to help control the remaining unassigned poles.

In the last section it will be shown that real and complex conjugate values can also be considered and that the Main Lemma continues to hold.

Remark: It is interesting to note how the number of assignable poles increases as a function of q, the order of the compensator. Assuming that $m|n$ and since "generically" the observability indices of the plant are all equal to $\mu = \frac{n}{m}$, we see the following:

If q=0, m poles are arbitrarily assigned.

q=1, 2m+1 poles are arbitrarily assigned

$\vdots$

$q=\mu-1$, $\mu m + \mu - 1 = n+\mu-1$ poles are arbitrarily assigned, which means that all closed loop poles can be arbitrarily assigned. A similar result when $q=\mu-1$ has been obtained earlier by Brasch and Pearson 1970 using a different approach.

An algorithm for constructing the solution involves expressing P in a right coprime representation and solving a linear equation over the reals (or complexes) since $\underline{y}$ from (3.2)

$$\underline{y} = -[s_1^i, s_2^i, \ldots, s_k^i](\bar{S}_{q+1} \cdot Q)^{-1} - \alpha Q(\bar{S}_{q+1}Q)^{-1} .$$

Comparing this procedure with the one suggested by Antsaklis and Wolovich 1970 for the mx1 case one can see that for a compensator of order q, using the present method $(m+1)q + m$ poles are assigned, whereas with the earlier one $2q + m$ are assigned. It should also be pointed out that the method used there is different than the present one in this respect as well, in that initially P is augmented by q integrators and then constant output feedback is used to close the loop.

The example below helps to illustrate and clarify the procedure.

Example 1

Let $m=2$, $\ell=1$, $n=6$.

$$d = s^6 + s^2 + 2 \qquad N = \begin{bmatrix} 1 \\ 0 \end{bmatrix} s^5 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} s^4 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} s^3 + \begin{bmatrix} 1 \\ 0 \end{bmatrix} s + \begin{bmatrix} 2 \\ 1 \end{bmatrix}$$

a) Let $q=0$, i.e. a constant compensator. The result suggests that 2 poles can be arbitrarily assigned.

Let $s_1 = -1$, $s_2 = -2$.

The compensator used is given by

$$x = 1 \qquad Y = [y_1, y_2]$$

and y_1, y_2 are obtained as the solution of

$$[y_1, y_2] \cdot \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 2 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix} \underbrace{\begin{bmatrix} s_1^5 & s_2^5 \\ s_1^4 & s_2^4 \\ s_1^3 & s_2^3 \\ s_1^2 & s_2^2 \\ s_1 & s_2 \\ 1 & 1 \end{bmatrix}}_{Q} = -[s_1^6, s_2^6] - [0, 0, 0, 1, 0, 2] \cdot Q$$

Computing the solution yields

$$y_1 = \frac{17}{16} \quad , \quad y_2 = -4$$

and the compensator

$$C = [\tfrac{17}{16} \quad , \quad -4].$$

b) Let q=1. Then 5 poles can arbitrarily be assigned.

Let $s_1 = -1$, $s_2 = -1.5$, $s_3 = -2$, $s_4 = -2.5$, $s_5 = -3$.

The compensator used is given by

$$x = s + x_1 \quad , \quad Y = [y_1, y_2]s + [y_3, y_4]$$

It is obtained as the solution of:

$$[y_1, y_2, x_1, y_3, y_4] \cdot \begin{bmatrix} 1&0&0&0&1&2&0 \\ 0&1&1&0&0&1&0 \\ 1&0&0&0&1&0&2 \\ 0&1&0&0&0&1&2 \\ 0&0&1&1&0&0&1 \end{bmatrix} \underbrace{\begin{bmatrix} s_1^6 & s_2^6 & s_3^6 & s_4^6 & s_5^6 \\ s_1^5 & s_2^5 & s_3^5 & s_4^5 & s_5^5 \\ s_1^4 & s_2^4 & s_3^4 & s_4^4 & s_5^4 \\ s_1^3 & s_2^3 & s_3^3 & s_4^3 & s_5^3 \\ s_1^2 & s_2^2 & s_3^2 & s_4^2 & s_5^2 \\ s_1 & s_2 & s_3 & s_4 & s_5 \\ 1 & 1 & 1 & 1 & 1 \end{bmatrix}}_{Q} =$$

$$= - [s_1^7, s_2^7, s_3^7\ s_4^7\ s_5^7] - [0, 0, 0, 1, 0, 2, 0]Q .$$

Computing the solution yields

$x_1 = 5.242544771594,$

$y_1 = 3.436523038651$ $\quad y_2 = 24.13305548336$

$y_3 = 2.405828042428$ $\quad y_3 = 7.162876396989$

and the compensator

$$C = \left[\frac{y_1 s + y_3}{s + x_1} \ , \ \frac{y_2 s + y_4}{s + x_1}\right] .$$

c) Let $q=2$. Then all 8 poles of the closed loop system can be assigned.

Let $s_1 = -1$, $s_2 = -1.1$, $s_3 = -1.2$, $s_4 = -1.3$, $s_5 = -1.4$, $s_6 = -1.5$, $s_7 = -1.6$, $s_8 = -1.7$.

The compensator which accomplishes this is given by $C = x^{-1}Y$

$$x = s^2 + x_1 s + x_2 \quad Y = [y_1, y_2]s^2 + [y_3, y_4]s + [y_5, y_6]$$

where

$x_1 = 275.0363186229 \quad , \quad x_2 = 63.27143983832$

$y_1 = -335.7496784115 \quad , \quad y_2 = 68.68243973376$

$y_3 = -91.31887959451 \quad , \quad y_4 = -41.2992798809$

$y_5 = -5.765759955678 \quad , \quad y_6 = 17.61171990276$

4. Multiple-Input, Multiple-Output Case

If P is an $m \times \ell$ $(m \geq \ell)$ strictly proper transfer function of McMillan degree n and equal controllability indices λ, (i.e. $n = \ell\lambda$) it can be expressed in the right coprime representation:

$$P = N_{RP} D_{RP}^{-1}$$

where

$$D_{RP} = Is^{\lambda} + D_{\lambda-1} s^{\lambda-1} + \ldots + D_o \qquad (4.1)$$

$$N_{RP} = \begin{bmatrix} N_{\lambda-1} \\ K_{\lambda-1} \end{bmatrix} s^{\lambda-1} + \ldots + \begin{bmatrix} N_o \\ K_o \end{bmatrix} .$$

D_i, N_i are $\ell \times \ell$ matrices, K_i are $(m-\ell) \times \ell$ matrices. Now if an $\ell \times m$ proper compensator $C = X^{-1} Y$ is used of the form

$$X = X_q s^q + X_{q-1} s^{q-1} + \ldots + X_o$$

$$Y = Y_q s^q + Y_{q-1} s^{q-1} + \ldots + Y_o$$

(X_i are $\ell \times \ell$, Y_i are $\ell \times m$), and X, Y are left coprime then the closed loop characteristic polynomial is given by

$$\phi(s) = \det(\underbrace{XD_{RP} + YN_{RP}}_{= \Phi}) .$$

If one uses Sylvester Resultants to express the relationships $XD_{RP} + YN_{RP}$ one has the following:

$$[X_q, Y_q, \ldots, X_o, Y_o] \begin{bmatrix} I, & D_{\lambda-1}, & D_{\lambda-2}, & \ldots, & D_o, & 0, & \ldots, & 0 \\ 0 & N_{\lambda-1}, & N_{\lambda-2}, & & N_o, & 0, & \ldots, & 0 \\ & & & \cdots & & & & \\ 0 & K_{\lambda-1}, & K_{\lambda-2}, & & K_o, & 0, & \ldots, & 0 \\ & & & & \vdots & & & \\ & & & & \ddots & & & \\ & & & I & D_{\lambda-1}, & D_{\lambda-2}, & \ldots, & 0 \\ & & & 0 & N_{\lambda-1}, & N_{\lambda-2}, & \ldots, & 0 \\ & & & 0 & K_{\lambda-1}, & K_{\lambda-2}, & \ldots, & 0 \end{bmatrix} \quad \begin{array}{c} q+1 \text{ block} \\ \text{rows} \\ (4.2) \end{array}$$

$$= [\Phi_{\lambda+q}, \Phi_{\lambda+q-1}, \ldots, \Phi_o] ,$$

where $$\Phi = \Phi_{\lambda+q} s^{\lambda+q} + \ldots + \Phi_o, \text{ is } \ell x \ell.$$

Now since $\phi(s) = \det \Phi$, it is evident that in general the coefficients of $\phi(s)$ are non-linear expressions in the compensator parameters. This is precisely where many difficulties concerning the pole assignment problem lie. One way of proceeding is to find ways of exploiting this non-linear structure. In this paper a successful approach is presneted which "avoids" the non-linear analysis. The problem is formulated in such a way that the nonlinear structure is forced to become linear.

Suppose that the compensators under consideration are restricted to have the following structure.

$$X = \underbrace{\begin{bmatrix} 1,0,0,0 \dots 0 \\ 0 & \\ \vdots & 0_{\ell-1} \\ 0 & \end{bmatrix}}_{X_q} s^q + \underbrace{\begin{bmatrix} {}_{q-1}x_1, 0 \dots 0 \\ 0 & \\ \vdots & 0_{\ell-1} \\ 0 & \end{bmatrix}}_{X_{q-1}} s^{q-1} + \dots + \underbrace{\begin{bmatrix} {}_0x_1, 0 \dots 0 \\ 0 & \\ \vdots & I_{\ell-1} \\ 0 & \end{bmatrix}}_{X_0} \tag{4.3}$$

$$Y = \underbrace{\begin{bmatrix} {}_qy_1, {}_qy_2, \cdots {}_qy_\ell \cdots {}_qy_m \\ \vdots & 0_{\ell-1} & 0 \\ 0 & & \end{bmatrix}}_{Y_q} s^q + \dots + \underbrace{\begin{bmatrix} {}_0y_1, {}_0y_2, \cdots {}_0y_\ell, \cdots {}_0y_m \\ \vdots & I_{\ell-1} & 0 \\ 0 & & \end{bmatrix}}_{Y_0}$$

Therefore

$$X = \begin{bmatrix} x(s), 0, \dots 0 \\ 0 & \\ \vdots & I_{\ell-1} \\ 0 & \end{bmatrix} \qquad Y = \begin{bmatrix} y_{11}(s), y_{12}(s), \dots y_{1\ell}(s), \dots y_{1m}(s) \\ 0 & & \\ \vdots & I_{\ell-1} & 0 \\ 0 & & \end{bmatrix} .$$

By construction $C = X^{-1}Y$ is proper.

Now if $\underline{y}(s)$ is the first row of Y, $(d_{11}(s), d_{12}(s), \dots, d_{1\ell}(s))$ the first row of D_{RP}, $\underline{n}_j(s)$ the j^{th} column of N_{RP} and $(\phi_{11}(s), \phi_{12}(s), \dots \phi_{1\ell}(s))$ the first row of ϕ one has the following:

$$\begin{aligned} x(s)d_{11}(s) + \underline{y}(s)\underline{n}_1(s) &= \phi_{11}(s) \\ x(s)d_{12}(s) + \underline{y}(s)\underline{n}_2(s) &= \phi_{12}(s) \\ &\vdots \\ x(s)d_{1\ell}(s) + \underline{y}(s)\underline{n}_\ell(s) &= \phi_{1\ell}(s) . \end{aligned}$$

Let $\bar{\Phi}$ be the $(\ell-1)x\ell$ matrix which contains the remaining $\ell-1$ rows of Φ. Then

$$\bar{\Phi} = [0, I_{\ell-1}]s^{\lambda} + (\bar{D}_{\lambda-1}+\bar{N}_{\lambda-1})s^{\lambda-1} + \ldots + (\bar{D}_0+\bar{N}_0)$$

where $\bar{D}_i$, $\bar{N}_i$ are obtained from D_i, N_i by removing the first row.

Now since the closed loop characteristic polynomial is $\phi(s) = \det\Phi$ one can easily see by expanding the first row that:

$$\phi(s) = \phi_{11}(s)\Delta_{11} + \phi_{12}(s)\Delta_{12} + \ldots + \phi_{1\ell}(s)\Delta_{1\ell} \qquad (4.4)$$

(Δ_{1i} is the appropriate $(\ell-1)x(\ell-1)$ minor of Φ). The coefficients of $\phi(s)$ are linear expressions in the compensator parameters. This relationship is made more explicit in the following:

$$\phi(s) = x(s)(\underbrace{d_{11}(s)\Delta_{11} + \ldots + d_{1\ell}(s)\Delta_{1\ell}}_{d}) + \underline{y}(s)(\underbrace{\underline{n}_1(s)\Delta_{11} + \ldots + \underline{n}_\ell(s)\Delta_{1\ell}}_{= N})$$

$$= x(s)d + \underline{y}(s)N. \qquad (4.5)$$

Since d is a polynomial of degree $\lambda\ell=n$, N an mx1 vector of degree $\lambda\ell-1=n-1$, x(s) a polynomial of degree q and $\underline{y}(s)$ a 1xm vector of degree q this fits precisely the formulation used in the analysis of mx1 systems. Namely (4.5) can be expressed as:

$$[1, Y_q, x_{q-1}, \ldots, x_0, Y_0]S_{q+1}(d,N) = [1, \Phi_{n+q}, \ldots \Phi_0] \qquad (4.6)$$

This would indicate that the Main Lemma could somehow be used in obtaining a result for the $m\times\ell$ case. In a sense the original $m\times\ell$ system has been "transformed" (or reduced) to the mx1 system corresponding to (4.5). This "transformation" will play a crucial role in the proof of the upcoming

Theorem, and is therefore now made more precise.

Let the given transfer function (4.1) be parameterized as follows:

$$D_{RP} = \begin{bmatrix} 1 & & & 0 \\ & \cdot & & \\ & & \cdot & \\ 0 & & & 1 \end{bmatrix} s^{\lambda} + \underbrace{\begin{bmatrix} a_1, & \ldots a_{\ell} \\ a_{\lambda\ell+1}, & \ldots a_{(\lambda+1)\ell} \\ & \cdot \\ & \cdot \\ & \cdot \\ a_{(\ell-1)\lambda\ell+1} & \cdots \end{bmatrix}}_{D_{\lambda-1}} s^{\lambda-1} + \ldots + \underbrace{\begin{bmatrix} a_{(\lambda-1)\ell+1} & \cdots a_{\lambda\ell} \\ & a_{2\lambda\ell} \\ \cdot & \cdot \\ & a_{\ell\cdot\lambda\cdot\ell} \end{bmatrix}}_{D_0}$$

$$N_{RP} = \begin{array}{c} \ell\left\{\begin{array}{c} \\ \\ \\ \\ \end{array}\right. \\ \\ m-\ell\left\{\begin{array}{c} \\ \\ \\ \\ \end{array}\right. \end{array} \underbrace{\begin{bmatrix} a_{\lambda\ell^2+1} & \cdots & a_{\lambda\ell^2+\ell} \\ & \cdot & \\ & \cdot & \\ & \cdot & \\ a_{\lambda\ell^2+(\ell-1)\lambda\ell+1} & \cdots & \\ a_{2\lambda\ell^2+1} & \cdots & \\ & \cdot & \\ & \cdot & \\ & \cdot & \\ a_{2\lambda\ell^2+(m-\ell-1)\lambda\ell+1} & \cdots & \end{bmatrix}}_{N_{\lambda-1}} s^{\lambda-1} + \ldots + \underbrace{\begin{bmatrix} a_{\lambda\ell^2+(\lambda-1)\ell+1} & \cdots & a_{\lambda\ell^2+\lambda\ell} \\ & \cdot & \\ & \cdot & \\ & \cdot & \\ & \cdots & a_{\lambda\ell^2+\lambda\ell^2} \\ & \cdots & a_{2\lambda\ell^2+\lambda\ell} \\ & \cdot & \\ & \cdot & \\ & \cdot & \\ & \cdots & a_{2\lambda\ell^2+(m-\ell)\lambda\ell} \end{bmatrix}}_{N_0} \begin{array}{c} \left.\begin{array}{c} \\ \\ \\ \\ \end{array}\right\}\ell \\ \\ \\ \\ \\ \end{array}$$

Let the reduced transfer function obtained in (4.5) be parameterized as follows:

$$d = s^n + b_1 s^{n-1} + b_2 s^{n-2} + \ldots + b_n$$

$$N = \begin{bmatrix} b_{n+1} \\ b_{2n+1} \\ \vdots \\ b_{\ell n+1} \\ b_{(\ell+1)n+1} \\ \vdots \\ \\ \end{bmatrix} s^{n-1} + \begin{bmatrix} b_{n+2} \\ b_{2n+2} \\ \vdots \\ b_{\ell n+2} \\ b_{(\ell+1)n+2} \\ \vdots \\ \\ \end{bmatrix} s^{n-2} + \dots + \left.\begin{bmatrix} b_{n+n} \\ b_{3n} \\ \vdots \\ b_{(\ell+1)n} \\ b_{(\ell+2)n} \\ \vdots \\ b_{(m+1)n} \end{bmatrix}\right\} \ell$$

Let $\underline{a} = (a_1, a_2, \dots, a_{\lambda\ell(m+\ell)})$. It is clear from (4.5) that each b_j is a function of $\underline{a}$ and will be indicated as such $b_j(\underline{a})$ whenever necessary. It is in fact a polynomial in $\underline{a}$.

Let f: $R^{n(m+\ell)} \longrightarrow R^{n(m+1)}$ be defined as:

$$f(\underline{a}) = (b_1, b_2, \dots, b_{(m+1)n}) = \underline{b}$$

The function f therefore describes the "transformation" precisely. The structure of f will play a very important role in the proof of the main result where it will be required to show that the Jacobian of f, J_f is full rank at some point $\underline{a}$. The following Proposition addresses this issue.

Proposition. Let J_f be the Jacobian of f. There exists a point $\underline{a}$ (i.e. a specific transfer function of the type (4.1))such that $J_f(\underline{a})$ is full rank.

Proof:

The minors Δ_{ij} of Φ were introduced in (4.4).

Let

$$\Delta_{11} = s^{(\ell-1)\lambda} + h_{11}s^{(\ell-1)\lambda-1} + \dots + h_{1,(\ell-1)\lambda}$$

$$\Delta_{12} = \qquad h_{21}s^{(\ell-1)\lambda-1} + \dots + h_{2,(\ell-1)\lambda}$$

$$\vdots$$

$$\Delta_{1\ell} = \qquad h_{\ell 1}s^{(\ell-1)\lambda-1} + \dots + h_{\ell,(\ell-1)\lambda} \;.$$

Let A be the nxn matrix

$$A = \left[\begin{array}{cccc:cccc:c:cccc}
1 & 0 & & 0 & 0 & 0 & \cdots & 0 & & & 0 & & \\
h_{11} & h_{21} & & h_{\ell 1} & 1 & 0 & \cdots & 0 & & & & & \\
h_{12} & h_{22} & & h_{\ell 2} & h_{11} & h_{21} & & h_{\ell 1} & & & & & \\
\vdots & \vdots & \cdots & \vdots & \vdots & \vdots & & \vdots & & 1 & 0 & & 0 \\
 & & & & & & \cdots & & \cdots & h_{11} & h_{21} & \cdots & h_{\ell 1} \\
h_{1,(\ell-1)\lambda} & h_{2,(\ell-1)\lambda} & & h_{\ell,(\ell-1)\lambda} & & & & & & \vdots & \vdots & & \vdots \\
0 & 0 & & 0 & h_{1,(\ell-1)\lambda} & h_{2,(\ell-1)\lambda} & & h_{\ell(\ell-1)\lambda} & & & & & \\
0 & 0 & & 0 & 0 & 0 & & 0 & & & & & \\
\vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots & & & & & \\
0 & 0 & & 0 & 0 & 0 & & 0 & & h_{1,(\ell-1)\lambda} & h_{2,(\ell-1)\lambda} & \cdots & h_{\ell,(\ell-1)\lambda}
\end{array}\right]$$

λ-1

λ block columns.

Let $\underline{\alpha}_1 = (a_1, a_2, \dots a_{\lambda\ell})$ $\quad 1^{st}$ row of D_{RP}

$\underline{\alpha}_2 = (\alpha_{\lambda\ell+1}, \dots \alpha_{2\lambda\ell})$ $\quad 2^{nd}$ row of D_{RP}

$\vdots$

$\underline{\alpha}_{m+\ell} = (a_{n(m+\ell-1)+1} \cdots a_{n(m+\ell)})$ last row of N_{RP}.

Let

$$\underline{\beta}_1 = \begin{bmatrix} b_1 \\ b_2 \\ \vdots \\ b_n \end{bmatrix}, \quad \underline{\beta}_2 = \begin{bmatrix} b_{n+1} \\ \vdots \\ b_{2n} \end{bmatrix}, \quad \dots \quad \underline{\beta}_{m+1} = \begin{bmatrix} b_{mn+1} \\ \vdots \\ b_{(m+1)n} \end{bmatrix}$$

Let

$$\bar{A} = \begin{bmatrix} 0 & 0 & \cdots & 0 \\ 0 & 0 & & 0 \\ A & & & \\ & A & & 0 \\ & & \ddots & \\ & 0 & & A \end{bmatrix}, \text{ an } (\ell+1)n \times (\ell-1)n \text{ matrix}$$

$$\bar{\bar{A}} = \begin{bmatrix} A & & 0 \\ & \ddots & \\ 0 & & A \end{bmatrix}, \text{ an } (m-\ell)n \times (m-\ell)n \text{ block diagonal matrix.}$$

Let B be an $(\ell+1)n \times (\ell-1)n$ matrix and E some $(m-\ell)n \times (\ell-1)n$ matrix. As will be seen very shortly the structure of B and E is not needed for the proof.

Then J_f is given by

$$J_f = \left[\begin{array}{c:c:c:c:c} \begin{matrix} A \\ \\ 0 \end{matrix} & B & \begin{matrix} 0 \\ A \\ 0 \end{matrix} & \bar{A} + B & 0 \\ \hdashline 0 & E & 0 & E & \bar{\bar{A}} \end{array}\right]$$

which can be transformed by a similarity transformation to:

$$\left[\begin{array}{c:cccc} & A & & & \\ B & & A & & 0 \\ \hdashline & & & \ddots & \\ E & & 0 & & \\ \hdashline & & & & A \end{array}\right].$$

coprime.

with a block diagonal (square) matrix with A's on the diagonal.

It is easily seen that for the point $\underline{a}$ defined by:

$$D = Is^{\lambda} \quad , \quad N = \begin{bmatrix} 0 & -1 & 0 & \cdots & 0 \\ 0 & 0 & -1 & & 0 \\ \vdots & & & \vdots & \\ & & & & -1 \\ 1 & 1 & 1 & & 1 \\ & & 0 & & \end{bmatrix}$$

A is full rank and so therefore is J_f. This completes the proof of the Proposition.

The stage has now been set for stating and prooving the main result of this paper.

Theorem

Let $P = N_{RP}D_{RP}^{-1}$ be an $m\times\ell$ $(m\geq\ell)$ strictly proper transfer function where

$$D_{RP} = Is^{\lambda} + D_{\lambda-1}s^{\lambda-1} + \ldots + D_0$$

$$N_{RP} = \begin{bmatrix} N_{\lambda-1} \\ K_{\lambda-1} \end{bmatrix} s^{\lambda-1} + \ldots + \begin{bmatrix} N_0 \\ K_0 \end{bmatrix}.$$

Let $m|n$ and $q\geq 0$ a fixed integer. Let $k = (q+1)m + q \leq n + q = i$, $\phi(s)$ the closed loop characteristic polynomial and define:

$$W = \left\{(N_{RP}, D_{RP}) \in R^{(m+\ell)n} \,\middle|\, D_{RP} = Is^{\lambda} + D_{\lambda-1}s^{\lambda-1} + \ldots + D_0,\ N_{RP} = \begin{bmatrix} N_{\lambda-1} \\ K_{\lambda-1} \end{bmatrix} s^{\lambda-1} + \ldots + \begin{bmatrix} N_0 \\ K_0 \end{bmatrix}\right\}$$

$$S = \{(s_1, s_2, \ldots, s_k) \in R^k \,|\, s_i \text{ real}\}$$

$$Z = \left\{(N_{RP}, D_{RP}, \underline{s}) \in R^{(m+\ell)n} \times R^k \,\middle|\, \begin{array}{l}\text{For which there exists a proper compensator of} \\ \text{order } q \text{ such that } s_1, s_2, \ldots, s_k \text{ are roots of } \phi(s)\end{array}\right\}$$

Then Z is a generic subset of $R^{(m+\ell)n} \times R^k$.

<u>Remark</u>: The requirement that $m|n$ is introduced merely for convenience.

<u>Proof</u>:

From (4.6) let $\underline{y} = [Y_q, x_{q-1}, \ldots, x_0, Y_0]$. Then as in the mx1 case

$$\underline{y} \cdot \overline{S}_{q+1}(d,N)Q = -[s_1^{\,i}, s_2^{\,i}, \ldots, s_k^{\,i}] - \alpha \cdot Q \,. \tag{4.7}$$

This follows directly from the reformulation of the $m \times \ell$ problem as an mx1 problem. $\overline{S}_{q+1}(d,N)$ is a matrix whose entries are polynomials in $\underline{a}$. Now the set $E \subseteq R^{(m+\ell)n} \times R^k$ for which a solution to 4.7 exists and is such that the corresponding $x(s)$, $\underline{y}(s)$ are left coprime is a Zariski open set. For it to be generic it must be shown to be non-empty. The Main Lemma guarantees this to be true for almost all $\underline{b} \times \underline{s}$. Since there exists an $\underline{a}$ (by the Proposition) for which $J_f(\underline{a})$ is full rank, then there exists an open set U such that $f(\underline{a}) \in U$ and $f(R^{(m+\ell)n}) \supseteq U$, (by the inverse function theorem Luenberger 1969). This means that E contains at least one point. This completes the proof of the Theorem.

The Theorem suggests that for "almost all" $m \times \ell$ transfer functions of McMillan degree n (and equal controllability indices $\lambda = \frac{n}{\ell}$) and for "almost all" $\underline{s}$, $(k = (q+1)m + q)$ there exists a proper compensator of order q such that $s_1, s_2, \ldots, s_k$ are k roots of the closed loop characteristic polynomial $\phi(s)$. As in the mx1 case some roots may be left unassigned. In the multi-input case the possibility does exist for introducing additional parameters in the compensator that can be used to control the remaining roots.

One such possibility is to modify the original compensator structure. Let $C = X^{-1}Y$,

$$X = \begin{bmatrix} x(s), 0, \ldots, 0 \\ 0 & \\ \vdots & I_{\ell-1} \\ & \end{bmatrix} \qquad Y = \begin{bmatrix} y_1(s), y_2(s), \ldots, y_\ell(s), \ldots, y_m(s) \\ 0 & & \\ \vdots & L & 0 \\ 0 & & \end{bmatrix}$$

where L is an $\ell-1 \times \ell-1$ constant matrix containing $(\ell-1)^2$ free parameters comprising a vector $\underline{c}$. The compensator $C = X^{-1}Y$ is still proper of order q. Following a similar proof to the one given one can show that k poles can be arbitrarily assigned for almost all $\underline{c} \times \underline{a} \times \underline{s} \in R^{(\ell-1)^2} \times R^{(m+\ell)n} \times R^k$. This means that for "almost all" choices of $\underline{a}$ and $\underline{s}$ a proper compensator of the form given above, which is parameterized by $\underline{c}$, (valid for "almost all" $\underline{c}$), can be constructed which assigns k poles of the closed loop system to $\underline{s} = (s_1,s_2,\ldots,s_k)$. The freedom afforded by the presence of these parameters can then be used to "control" the location of the remaining unassigned poles. An illustrative example is given in section 5.

Remark: As in the mx1 case the number of assignable poles increases as a function of q in such a way that if $q = \mu-1$ then all the closed loop poles are arbitrarily assigned. Brasch and Pearson 1970 show, in an entirely different way, that for a controllable observable system adding a $\mu-1$ order compensator is sufficient to ensure arbitrary pole assignment.

5. Complex Poles

The concern thus far has been the arbitrary assignment of a number of real poles. The results remain valid for the case when real and complex conjugate roots are desired. It is evident that generically a solution to (3.1) will exist. The only requirement is the invertibility of $\overline{S}_{q+1} \cdot Q$. Since the solution contains the compensator parameters the critical issue is to show that the unique solution is real. This issue is addressed in the following Lemma.

Lemma.

Let $\underline{s} = (s_1, s_2, s_3, s_4, \ldots, s_{2j-1}, s_{2j}, s_{2j+1}, \ldots, s_k)$

be a set of k-2j real and 2j complex conjugate values, $((s_1,s_2), (s_3,s_4) \ldots (s_{2j-1},s_{2j})$ are j complex conjugate pairs and $s_{2j+1}, \ldots, s_k$ k-2j real values.

The unique solution $\underline{y}$ of (3.2)

$$\underline{y} \cdot \overline{S}_{q+1}(d,N) \cdot Q = - [s_1^i, s_2^i, \ldots, s_k^i] - \alpha \cdot Q$$

(whenever it exists) is real.

Proof:

Let the i^{th} row of $\overline{S}_{q+1}$ be thought of as the coefficients of polynomial ϕ_i. Then

$$\overline{S}_{q+1} \cdot Q = \begin{bmatrix} \phi_1(s_1), \phi_1(s_2), & \ldots, & \phi_1(s_k) \\ \phi_2(s_1), \phi_2(s_2), & \ldots, & \phi_2(s_k) \\ & \vdots & \\ \phi_k(s_1), \phi_k(s_2), & \ldots, & \phi_k(s_k) \end{bmatrix} = M .$$

Since

$$\underline{y} = -[s_1^i, s_2^i, \ldots, s_k^i]M^{-1} - \alpha \cdot QM^{-1}$$

then $\underline{y}$ will be real if

$$[s_1^t, s_2^t, \ldots, s_k^t]M^{-1} = [u_1, u_2, \ldots, u_k] \quad \text{is real for}$$

every integer $t \geq 0$.

If M_{ij} is the i,j minor of M then

$$u_j = \frac{[(-1)^{j+1}s_1^tM_{j1} + (-1)^{j+2}s_2^tM_{j2} + \ldots + (-1)^{j+k}s_k^tM_{jk}]}{(-1)^{j+1}\phi_j(s_1)M_{j1} + (-1)^{j+2}\phi_j(s_2)M_{j2} + \ldots + (-1)^{j+k}\phi_j(s_k)M_{jk}}$$

where det M is expanded using the j^{th} row.

It is not difficult to see that $u_j = u_j^*$ where * indicates complex conjugate. This means that u_j and therefore $\underline{y}$ is real.

Using this result one can easily see that the Main Lemma and Theorem still hold if $\underline{s}$ contains real and complex conjugate values.

The following example helps to illustrate the pole assignment method in the multi-input multi-output case.

<u>Example 2</u>

Let m=2, ℓ=2, n=4 and

$$D_{RP} = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} s^2 \qquad N_{RP} = \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix} s + \begin{bmatrix} 0 & -1 \\ 1 & 1 \end{bmatrix}.$$

Using the modified compensator structure given in (4.8),

$$X = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \qquad Y = \begin{bmatrix} y_1 & y_2 \\ 0 & c \end{bmatrix},$$

this transfer function is reduced to the 2x1 transfer function

$$d = s^4 + cs^2 \qquad N = \begin{bmatrix} 0 \\ 1 \end{bmatrix} s^3 + \begin{bmatrix} 0 \\ 1 \end{bmatrix} s^2 + \begin{bmatrix} c \\ 0 \end{bmatrix} s + \begin{bmatrix} c \\ 0 \end{bmatrix} .$$

The results suggest that with a constant compensator (q=0), 2 poles can be assigned.

Let $\quad s_1 = (-1 + j2) \quad , \quad s_2 = (-1 - j2)$.

The compensator parameters y_1, y_2 are given as the solution of

$$[y_1, y_2] = \begin{bmatrix} 0 & 0 & c & c \\ 1 & 1 & 0 & 0 \end{bmatrix} \begin{bmatrix} s_1^3 & s_2^3 \\ s_1^2 & s_2^2 \\ s_1 & s_2 \\ 1 & 1 \end{bmatrix} = - [s_1^4, s_2^4] - [0,c,0,0] \begin{bmatrix} s_1^3 & s_2^3 \\ s_1^2 & s_2^2 \\ s_1 & s_2 \\ 1 & 1 \end{bmatrix}$$

Computing the solution yields

$$y_1 = \frac{(c-3)25}{8c} , \qquad y_2 = \frac{7+3c}{8} ,$$

and the compensator in <u>parametric</u> form

$$C = \begin{bmatrix} \frac{(c-3)25}{8c} & \frac{7+3c}{8} \\ 0 & c \end{bmatrix} .$$

This compensator makes the closed loop characteristic polynomial equal to

$$\phi(s) = (s^2 + 2s + 5)(s^2 + \tfrac{3}{8}(c-3)s + \tfrac{5}{8}(c-3)).$$

One can easily see that for "almost all" choices of c(i.e. $c \neq 0$) the compensator makes $-1 + j2$ and $-1 - j2$ two of the closed loop poles. In this simple example the remaining two roots can be explicitly expressed as functions of c.

$$s_{3,4} = \frac{-3(c-3) \pm \sqrt{9c^2 - 214c + 561}}{16}$$

In particular for $c > 3$, s_3 and s_4 are guaranteed to be stable. Had one used the compensator structure suggested in (4.3), $c=1$, it would correspond to $s_3 = 1.5542476$, $s_4 = -.8042476$. which includes an undesirable pole.

The above suggestion becomes a very powerful tool in that the compensator C is given, parameterized by $\underline{c}$, that assigns k of the closed loop poles. Since the remaining poles in general depend on $\underline{c}$, they can in turn be controlled.

6. Conclusions

Using an approach involving Sylvester Resultants it is demonstrated that generically min(n+q, (q+1)m+q) closed loop poles can be arbitrarily assigned with an output fedback compensator of order q. It is further suggested how the locations of the remaining unassigned poles could be controlled. The approach is different than the ones followed by Antsaklis and Wolovich 1977, Brasch and Pearson 1970, Kimura 1975, Davison and Wang 1975. For the appropriate cases the result is an improvement of the earlier result (Antsaklis and Wolovich 1977, Kimura 1975) for dynamic output feedback. The method of solution can be easily programmed on a digital computer.

It is my belief that the results in the multivariable case can be strengthened by exploiting more effectively the compensator structure.

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